Understanding and Enriching Problem Solving in Primary Mathematics

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Patrick Barmby, David Bolden
& Lynn Thompson

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Contents

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Meet the authors

Patrick Barmby is a lecturer in primary mathematics at Durham University and is the Programme Director for the BA Primary Education course at Durham. Patrick's area of research is in primary mathematics, with a particular focus on developing children's understanding in mathematics. He has also carried out research projects on developing teachers' knowledge in primary mathematics. For him, the best parts of teaching on the Primary Education degree at Durham University are not only influencing student teachers' practice in primary mathematics, but also being influenced himself by the wealth of experiences and expertise from the classroom that students bring to the sessions.

David Bolden is a Lecturer in Mathematics Education at Durham University where he teaches on the BA Primary Education, PGCE Primary Education and MSc Mathematics Education courses. His research interests include creativity in mathematics and teacher epistemologies, particularly in relation to primary mathematics.

Lynn Thompson is an educational consultant and founder of North East School Support (NESS) supporting schools to raise standards and improve the teaching and learning of mathematics. She also teaches part time on the Primary BA and PGCE courses at Durham University.

1 Introduction

Key issues in this chapter

- This introductory chapter begins by outlining the reasons for writing this book and why we feel that it is important for teachers to be clear about the teaching and learning of problem solving.
- It sets out a short history of the ways in which problem solving has been conceived and incorporated into the different iterations of the primary curricula, and why this book is timely.
- Having justified 'why' we have written this book, the chapter outlines 'what' the book contains in terms of problem solving and 'how' we have approached the topic.

Introduction

This book is about problem solving in the primary mathematics classroom. We should state at the outset that we are acutely aware that there exist a number of very good books already published on the subject (for instance, Polya, 1957, Burton, 1984, and Mason, Burton & Stacey, 1985, to name just a few), but we think we have something to add. Reflecting upon the teaching, understanding and use of problem solving skills, however, we felt there was now a need for a complementary yet different style of book which would not only inform teachers, but would directly support their teaching in the classroom. This book is akin to a high-quality training course; a course that you can complete at your own leisure and that will undoubtedly improve pedagogical practice and impact upon children's understanding, engagement and achievement. This book is an amalgamation of previous and current research findings that will question your thinking and test your subject knowledge. It unpicks the problem-solving process in detail and discusses classroom implications that relate directly to the requirements of the national curriculum (DfE, 2013). We present chapters that ask questions like *What is the nature of a problem and of the problem solving process*? *What type of characteristics should genuine problems exhibit? And what does all this mean for* *the mathematics classroom, the pupil and the teacher?* Each chapter deals with a different part of the process as we view it and provides an up-to-date review of the research and theory relating to that particular aspect. In addition, each chapter also provides concrete pedagogical examples of those aspects in action in the primary classroom. We encourage current trainees, NQTs and established teachers to question their own practice in the light of what we discuss in this book. We acknowledge that developing and incorporating problem solving as we view it in the classroom is not always easy, but we think that the results will be worth the effort, both for the teacher and the pupil.

Why now?

Problem solving has always been viewed by many in the field of mathematics education to be at the very heart of mathematics.We think the book is particularly timely because problem solving now seems to be taking on greater importance in the eyes of policymakers, for whom this certainty has not always existed. For instance, the government's education watchdog, OFSTED, recently identified the planning and assessing of problem solving as an area of weakness in trainee teachers' practice, and suggested this as an area for improvement (OFSTED, 2013). As a result, we are in a situation where there appears to be a large degree of consensus concerning the importance of problem solving in the teaching and learning of mathematics. It is now viewed as important by both academics and policy makers and we think this is encouraging. We hope this book will build on and further contribute to that consensus while also helping teachers to engage with the central ideas.

A short 30-year history of problem solving in the UK

The Cockroft report

It was the Cockroft report in the UK (Cockroft, 1982) which re-established problem solving as the heart of effective learning and teaching in mathematics, although many in the field of mathematics education had never swayed from that belief. The Cockroft report was commissioned by the UK government at a time when concerns were being raised that young pupils in primary schools were not developing the necessary mathematical skills and understanding and that teachers were relying too heavily on published schemes in their teaching. The report argued that *'the ability to solve problems is at the heart of mathematics'* (Cockcroft, 1982, p. 249). It included in its list of recommendations that problem solving be better integrated into the primary mathematics curriculum including more investigational work and that mathematical knowledge be better applied to everyday situations.

The national curriculum

Although much good work continued in the following years, including the publication of influential books by Burton (1984) and Mason et al, (1985) mentioned above, by the late 1980s there still existed a concern, predominantly but not exclusively in industry, that pupils in English schools were under-performing. The Conservative government at the time attempted to address that concern by adopting a ‘back to basics’ approach which set about prescribing what pupils should be taught and when. In the first iteration of this national urriculum in 1988–89 (DfES, 1989), problem solving was encapsulated in the term ‘using and applying’ mathematics. This was useful in that in it seemed to distinguish between the learning of mathematical facts and skills, and the more important and overarching processes of applying those facts and skills in unfamiliar situations. These processes included enquiring, representing, reasoning, and communicating. However, although many were encouraged by this new emphasis on problem solving, the way in which using and applying was positioned within and among the other aspects of the curriculum, eg as a separate attainment target, seemed to suggest it was not conceived as a process that should be applied across all areas of mathematics. Moreover, assessment of children’s progression – in all areas of mathematics – was to take the form of standardised tests, and high stakes tests at that, which were not and are not conducive to assessing children’s problem-solving processes.

In a subsequent iteration of the national curriculum in 1999 (DfEE, 1999a), ‘using and applying’ was no longer to be seen as a separate entity but was now incorporated into each of the other aspects of the primary mathematics curriculum. At about the same time, the National Numeracy Strategy was introduced (DfEE, 1999b). This set out a framework for teaching mathematics across the primary age range and offered teachers a great deal of structure, including the three-part numeracy lesson. However, the emphasis was on calculating and number work and not on problem solving per se, which was seen by many as an ‘add-on’ and not something that was integral to everyday mathematics.

In 2003 the Labour government set out a new vision for primary education with the publication of Excellence and Enjoyment: A Strategy for Primary Schools (DfES, 2003). Although it emphasised that pupils should be enthused by mathematics, it mentioned creativity more than problem solving or ‘using and applying’. The Primary National Strategy framework documents (DfES, 2006a; 2006b) re-emphasised the central importance of ‘using and applying’ by stating that it needed to be *‘embedded into the teaching and learning of mathematics’*, that it *‘become a regular part of children’s work…* [and that] *…problem solving should not be seen as a “Friday only” activity.’* (p.7). The Williams Review (DCSF, 2008) continued this closer examination of the mathematics curriculum, with a particular emphasis on the provision in primary schools. It argued that the curriculum on offer was *‘… by and large, well balanced’* but argued for *‘an increased focus on the “use and application”* of mathematics’ (p.4).

Reference on page 259 does not refer to this as the Williams Review. Is this OK?

Not referred to as the Rose Review in refer-ence on page 259. Does this matter?

The Rose Review (DCSF, 2009) was commissioned by the then Labour government to examine the entire primary curriculum and its main aim was to suggest a provision that was fit for purpose as we moved into the 21st century. It advocated mathematical understanding as one of six main areas of study with, again, an emphasis on *‘using and applying’*, but its recommendations came at the end of Labour’s term of office and were quickly discarded by the incoming Conservative government. All this was happening at a time when Ofsted was reporting general findings that pupils observed during inspections often had the ability to perform written calculations but were not so good at applying that knowledge in unfamiliar and/or real life situations. The report, Mathematics: Understanding the Score (Ofsted, 2008),

Please provide a full reference for the references section painted a gloomy picture in terms of children's ability to use and apply their mathematics knowledge:

> *'Too often, pupils are expected to remember methods, rules and facts without grasping the underpinning concepts, making connections with earlier learning and other topics, and making sense of the mathematics so that they can use it independently. The nature of teaching and assessment, as well as the interpretation of the mathematics curriculum, often combine to leave pupils ill equipped to use and apply mathematics. Pupils rarely investigate open-ended problems which might offer them opportunities to choose which approach to adopt or to reason and generalise.'*
>
> (Ofsted, 2008, p.5)

In light of its findings, the report went on to recommend that separate reporting of pupils' attainment in the area of *'using and applying mathematics'* be reintroduced as part of the statutory teacher assessments at the end of each key stage.

The present day

The advent of the new national curriculum (DfE, 2013) brings us to the present day. Here we see that using and applying is again regarded as an integral part of the curriculum and that pupils are expected to *'… solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.'* (p.3).

Much good work was conducted by those in the field of mathematics education during this time (eg NRICH) but the history of problem solving over the last 30 years is perhaps best summed up by Burkart and Bell (2007) in their useful historical account. They write that the history of problem solving in the UK *'… illustrates the way policy decisions, taken on plausible grounds by people with good intentions but limited understanding, frequently have unintended consequences that undermine the very goals they seek to advance'* (p.401). Consequently, the results in the classroom have, up until now, been patchy at best.

Primary teachers' experiences with problem solving

As the following chapters will illustrate, problem solving requires the solver to use a number of what we might call *'higher level skills'* (Burkart & Bell, 2007: p.395), eg patience, resilience, logical reasoning and a range of meta-cognitive skills, and so it is perhaps surprising that primary teachers, who are often not specialists in any particular subject domain, have not always felt comfortable with delivering this aspect of the curriculum. Our own research (Bolden, Harries & Newton, 2012), and the research of others, shows that primary teachers often view mathematics as an unchanging set body of knowledge to be taught and learned. They regard the primary aim of lessons as the learning of specific procedures, sometimes by rote, and ultimately the search for the one, and only one, right answer. Primary teachers may have sound mathematical subject knowledge in this respect but it can often be lacking in conceptual knowledge, that is the type of knowledge which allows them to make links

Is this the title referred to as 'Newton, 2012' in the references?

between what appear to be disparate areas of mathematics, ie to understand. We do not wish to suggest that rote learning is not important, it can be very useful in the right context, but we do wish to suggest that true problem solving requires a broader and richer range of skills.

There is sometimes great pressure on teachers to cover the content in the curriculum (Bolden & Newton, 2008) and this does not help them to develop rich problems with which pupils can explore mathematics. Creating rich problems and engaging in real problem solving takes time – both on the part of the teachers and the pupils – and the current and past curricula have left teachers feeling there is insufficient time and space for this to happen successfully.

How this book is organised

Given teachers' experiences with problem solving, this book sets out to clarify exactly what problem solving is, the processes involved in problem solving (including thinking and learning processes by pupils), and how problem solving and the variety of related processes can be incorporated into the mathematics classroom. The approach we have taken on problem solving in this book is to take a primary classroom perspective, rather than considering it from a more 'advanced' perspective related to degree, A-level or even secondary mathematics. The main reason for this is to show that, even with younger pupils, you can incorporate problem-solving approaches in your teaching. However, this does not mean that this book is relevant to primary teachers only. In fact, in putting forward our arguments throughout the book, we draw on research on problem solving across the age range of schooling. You might adapt the examples given for the pupils that you work with, but the important thing that we want you to take from this book is this clearer view of problem solving.

Therefore, in Chapter 2, the first thing we look at is what problem solving is. In doing so, we question what a 'problem' is, and then explore suggestions from research and practice on how we might suggest pupils tackle these problems, thereby clarifying the problem solving process. You will see from our discussions regarding this process that 'representing' the problem situation, and the mathematical reasoning or thinking involved, are crucial aspects of this process. Therefore, in Chapter 3, we examine this issue of representing problems, and in Chapter 4, we examine the process of reasoning and how we reason with problems. Strongly related to this issue of reasoning is the creative process involved in problem solving, and we examine creativity and problem solving in Chapter 5. Based on all the issues covered in Chapters 2 to 5, we then examine in Chapter 6 how we can assess the problem solving process, clearly an important consideration for teachers.

After Chapter 6, we look to broaden our discussions and to examine other activities and processes related to problem solving. Firstly in Chapter 7, we look at other *'open approaches'* to mathematics such as investigations and mathematical modelling, and also other approaches to teaching and learning such as the *'realistic mathematics education'* approach developed in the Netherlands. In Chapter 8, we examine the issues of enriching children's mathematical experiences and engaging them in the subject, and the role that approaches such as problem solving play in these issues. We conclude in Chapter 9 by reflecting on all the issues that we have discussed in the book, bringing the ideas together in a coherent whole. Throughout each subsequent chapter we ask you some *critical questions* which are designed to make you reflect on your own views of the issues we discuss and to explore your own practice. We also offer you some ideas for *taking it further* by presenting additional reading which we think might help you develop some of the ideas.

In reading this book, we would like you to keep the following quote from Richard Feynman, the Nobel Prize-winning theoretical physicist, in mind:

> *'It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong.'*
>
> Richard Feynman

We know that we have drawn on quite a bit of research in this book to clarify problem solving. e have done this in order to try and get a deeper insight into the issue. But more important than accepting the ideas from the research is how the ideas fit in with your experience, both in terms of doing problem solving and also teaching problem solving. Therefore, we hope you enjoy reflecting upon the ideas in this book and that it does indeed provide you with deeper insights into your experiences of problem solving.

Critical question

» *At the very outset of this book, what are your own views of problem solving? What do you understand by the term problem solving? What role does it play in your classroom or the classrooms you have observed?*

Taking it further

As identified in the text, Burkart and Bell (2007) provide a historical overview of problem solving in the UK context. In addition, Askew (1996) provides an overview of *'using and applying'* in relation to problem solving, but also explores the difficulties that teachers face in coming to understand and teach *'using and applying'* in the mathematics classroom.

Index